Comparative study of reliability-based optimum performance analysis of structure using TLCBD and TLCD

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Abstract

Tuned liquid column ball damper (TLCBD) is a very effective control system to mitigate the response of the structures. A comparative study of reliability-based optimization of TLCBD over Tuned liquid column damper (TLCD) in the stochastic earthquake has been assessed. This study has been performed to evaluate the performance of the structure in terms of probability failure.

The objective function is taken as the first passage probability of failure of the damper systems. A numerical study is considered to evaluate the probability of failure of the single degree of freedom (SDOF) modelled structure against different structural and damper parameters. The study shows that applying TLCBD-structure system provides more safety over the conventional TLCD-structure system.

Keywords: Tuned liquid column ball damper, stochastic earthquake, tuned liquid column damper, reliability-based design optimization.

Introduction

In recent time TLCD is effectively used in modern structures for mitigating its vibrations caused by the different dynamic forces (seismic[13], wind[32] and wave motion[18]). The ease of installation, low maintenance cost and stable performance under an extensive range of exciting frequencies made TLCD a better choice among other passive damping devices. TLCD usually consists of a U shaped cross-sectional tube filled with water where an orifice is placed at the centre of the horizontal part[16]. The optimal performance of TLCD was earlier determined by Chang et al[3] considering the different variables and their relationship with the various other parameters by minimizing the objective function.

In the process of minimizing the desired objective function, optimum LCD parameters can be obtained by searching an appropriate series of design variables over a possible permissible domain, well known as stochastic structural optimisation (SSO)[27].

The Root Mean Square (RMS) response (acceleration, velocity and displacement) and total building life cycle costs etc. are usually considered as objective functions in SSO. Chang et al[4] found out the optimum parameters (optimum head loss coefficient and optimum tuning ratio) for proper designing of TLCD under the case of SSO.

Jorge et al[14] researched to measure the efficiency of TLCD and defined the optimal parameters and their contribution to confirm the best performance. Ahadi et al[1] used multiple TLCD for mitigating the response of a ten-storey linear shear frame structure subjected to white noise seismic excitation. A bidirectional TLCD having reduced mass of water was proposed by Rozas et al.[24] They compared it with two independent orthogonal TLCDs for better controlling of the response of the structure. Several degrees of freedom (DOF) structures have been considered, analysed and verified by practical experimentation for the same and finally, a device was designed and developed to mitigate a six DOFs structure.

In order to improve the performance of TLCD, different kinds of modifications have been proposed on the existing TLCD as 1: combined spring and a viscous damper[9], 2: variable orifice[11], 3: magneto-rheological fluid in place of normal fluid[31]. Following these works, Al-Saif et al[2] conceptualized the design of TLCBD from existing TLCD by introducing a metal ball for the first time instead of the orifice in the horizontal part of the column. It was exposed that the metal ball works as a rolling orifice and performs better than immobile orifice.

Following the work of Al-Saif et al,[2] Chatterjee and Chakraborty[7] again carried out an analytical study on TLCBD and existing TLCD, both subjected to wave-induced vibration attached on an SDOF system. Parametric and optimised response study under wave force were investigated and better performance of TLCBD was assured over TLCD[7].

Gur et al[10] also performed a similar comparative study under stochastic and real earthquake loading conditions and robustness of TLCBD was further confirmed. Pandey and Mishra[22] designed a circular shaped TLCBD and they assessed the response of structure added with modified TLCBD, subjected under torsional coupled vibration generated due to wind.

Tanveer et al[30] studied the efficiency of TLCBD over TLCD, attached on a multi-storeyed structure. Both the systems were evaluated analytically and experimentally and TLCBD was found more suitable to mitigate the response of the structure subjected to harmonic and seismic excitation.

Further, tuned liquid column gas damper and tuned liquid column ball gas damper were introduced in this regard to improve the productivity TLCBD[12].

Furthermore, in our earlier work[21], it was also observed that if the excessive liquid movement in vertical column was restricted up to a certain limit, then the performance of TLCBD can be regulated for higher level of seismic excitation.

The structural safety and reliability against failure majorly depend upon the uncertain nature of the environmental loads, structural properties and important consideration in the design. Structural failure is defined as the condition of the structure under which certain strength or serviceability limit states are exceeded. Merely considering the performance of response reduction cannot be the prior importance for structure. Reliability and the safety of the structure are more useful than the minimisation of the response reduction[19].

In this context, the Reliability-Based Design Optimization (RBDO) was suggested by Papadimitriou et al[23] for passive dampers subjected to stochastic excitation with random parameters. RBDO was used for minimization of the failure probability of the system[28]. They worked on passive and active structural control applications for minimizing the probability of failure of the prime structure by optimizing the response control system.

For different passive devices, reliability assessment has been performed and analysed for proving the better performance. It has been observed that detailed literature is available on the reliability and safety for different types of passive vibration devices as base isolation system[8,33] and tuned mass damper[5], but, limited studies were found on TLCD[6].

Although, the reliability performance has equally paramount importance, due to the stochastic nature of the earthquake load, the performance study of TLCBD was done for minimizing the structural response only. Since TLCBD is a developed form of the existing TLCD, therefore, the present study is aimed to examine the reliability analysis of TLCBD system as compared to TLCD.

In this present study, an SDOF system has been considered as a primary structure attached with two different dampers separately (TLCD and TLCBD), for these two damper systems the stochastic response analysis is evaluated first and then with the help of first-passage reliability theory, the optimised probability of failure value of primary structure and corresponding optimum parameters of the damper are determined and eventually probability of failure values are compared and demonstrated with help of a numerical study.

Stochastic structural response analysis of TLCBD-structure system: In figure 1a, the schematic diagram of TLCBD is presented where a metal ball of spherical shape is placed instead of the orifice in the middle part of the damper. Whenever the damper-structure system is subjected to dynamic forces, the same translatory motion is transmitted to the liquid column tube[17]. The overall length (L) is expressed as $L = B_h + 2h$. The (B_h) represents the length of the liquid present in the horizontal tube and h signifies the vertical height of the liquid measured from central line of horizontal column, shown in figure 1a.

Because of that the liquid and ball present in the damper become exposed to the translatory movement. TLCBD is normally placed on the uppermost part of the structure, modelled as an SDOF system. The motion equation of the ball present at the horizontal part can be derived by using the Lagrangian formulation[10].

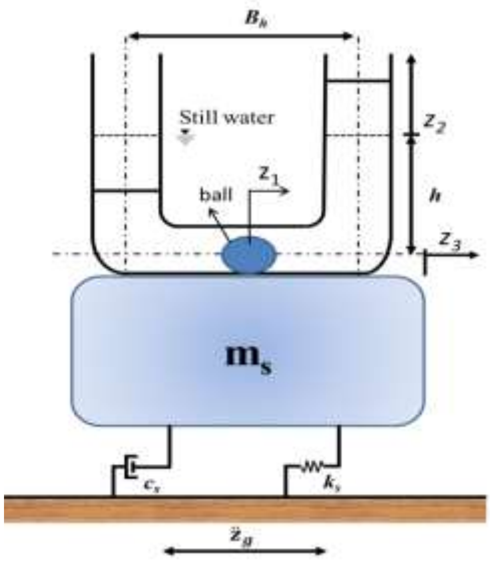

Figure 1a: Mechanical model of TLCBD-Structure system

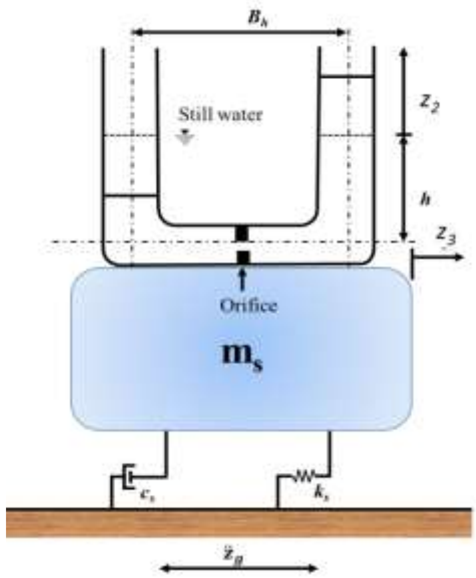

Figure 1b: Mechanical model of TLCD-Structure system

$$\left(m_b + \frac{I_b}{R_b^2}\right)\ddot{z}_1 + d_v\dot{z}_1 = \left(\frac{2m_l g R_{12}^2}{L}\right)z_2 + d_v\dot{z}_2 + \left(\frac{I_b}{R_b^2}\right)(\ddot{z}_3 + \ddot{z}_g) \quad (1)$$

The displacement of the ball is represented by z_1. Equivalent viscous damping of the ball (figure 1a) is denoted by:

$$d_v = 6\pi\nu R_b$$

where mass moment of inertia of the metal ball is represented as $I_b = 2m_b R_b^2/5$. The mass of the ball is denoted as m_b.

The kinematic viscosity of the liquid is signified as ν. R_b denotes the radius of the ball and the ball to tube diameter ratio is represented as R_{12}. z_3 signifies lateral displacement of structure with respect to ground. Displacement of the liquid is represented by z_2.

Single and double dot symbolise the velocity and acceleration of the same. The natural frequency (ω_l) of the liquid is presented as $\sqrt{2g/L}$ and the tuning ratio (γ) which is (ω_l/ω_s) of TLCD and TLCBD has been denoted as γ_l and γ_b respectively. $\ddot{z}_g$ is the seismic acceleration applied at the ground level of the structure. Gravitational acceleration has been expressed by g.

The motion equation for the liquid mass here is expressed in equation (2):

$$m_l\ddot{z}_2 + \left(\frac{2m_l g}{L}\right)z_2 + (2m_l\xi_l\omega_l)\dot{z}_2 = -pm_l(\ddot{z}_3 + \ddot{z}_g) \quad (2)$$

Here, ξ_l signifies the head loss coefficient implying the damping generated by the liquid. Here, p is the length ratio represented as (B_h/L). The density of the liquid is ρ_l.

The equation of motion of the structure here is expressed as:

$$\left(m_s + m_l + \frac{I_b}{R_b^2}\right)\ddot{z}_3 + 2m_s\xi_s\omega_s\dot{z}_3 + m_s\omega_s^2 z_3 = \left(\frac{I_b}{R_b^2}\right)\ddot{z}_1 - pm_l\ddot{z}_2 - \left(m_s + m_l + \frac{I_b}{R_b^2}\right)\ddot{z}_g \quad (3)$$

Here, the natural frequency and damping ratio of the structure are denoted by $\omega_s = \sqrt{k_s/m_s}$ and $\xi_s = c_s/2\sqrt{k_s m_s}$ respectively. where, damping, stiffness, mass of the primary structure are denoted by c_s, k_s, m_s. $\mu = (m_l/m_s)$ defines the mass ratio.

The equations (1), (2) and (3) are combined to express them in a matrix form as equation (4):

$$\begin{bmatrix} 1 & 0 & -q_1 \\ 0 & 1 & p \\ -\mu_1 & \mu p & (1+\mu+\mu_1) \end{bmatrix}\begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ 0 & 2\gamma_b\xi_s\omega_s & 0 \\ 0 & 0 & 2\xi_s\omega_s \end{bmatrix}\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{Bmatrix} + \begin{bmatrix} 0 & -k_1 & 0 \\ 0 & \omega_l^2 & 0 \\ 0 & 0 & \omega_s^2 \end{bmatrix}\begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = -\ddot{z}_g\{r\}\begin{bmatrix} 1 & 0 & -q_1 \\ 0 & 1 & p \\ -\mu_1 & \mu p & (1+\mu+\mu_1) \end{bmatrix} \quad (4)$$

$\{0 \quad 0 \quad 1\}^T$ is the value of the influence coefficient vector, $\{r\}$. The other abbreviation is used as follows, $q_1 = 2/7$, $\mu_1 = 2m_b/5m_s$, $c_1 = 45\nu/14R_b^2\rho_b$, $k_1 = 15\rho_l/14\rho_b R_b$ where ρ_b is the density of the ball.

To assess the response of TLCBD-structure system, at the foundation level of the structures, the ground motion having stochastic properties has been applied. The equations (4) can be written in a concise form for the system as:

$$[M]\{\ddot{Z}\} + [C]\{\dot{Z}\} + [K]\{Z\} = -[M]\{r\}\ddot{z}_g \quad (5)$$

The [C], [K], and [M] represent the combined damping, stiffness and mass matrices for the system.

To represent the stochastic excitation for a wide-ranging scope of pragmatic circumstances, a broadly accepted model for the stationary ground movement known as the Kanai-Tajimi model[15,25] has been used. Here, white noise which is acting at the bedrock portion is filtering through the filter which is signifying as a soil.

The filter equations are stated as:

$$\ddot{z}_f + 2\xi_g\omega_g\dot{z}_f + \omega_g^2 z_f = -\ddot{w} \quad (6)$$

$$\ddot{z}_g = -2\xi_g\omega_g\dot{z}_f - \omega_g^2 z_f \quad (7)$$

where $\ddot{w}$ denotes white noise intensity having power spectral density S_0. z_f represents the displacement of the ground. The damping ratio is represented as ξ_g and ω_g as the frequency of the soil. The introduction of the stochastic structural excitation has been formulated by substituting $\ddot{z}_g$ from equation (7) and incorporating it with equations (4) which is the dynamic equations of TLCBD-structure system. The augment state vector is used to introduce the state variables and can be expressed for the TLCBD-structure system as follows:

$$[Y] = [z_1, z_2, z_3, z_f, \dot{z}_1, \dot{z}_2, \dot{z}_3, \dot{z}_f]^T \quad (8)$$

The response of the structure can be evaluated by converting the above dynamic equation into the state space equations:

$$[\dot{Y}] = [A][Y] + \{r\}\ddot{z}_g \quad (9)$$

where $[A]$ is augmented matrix of size (8 x 8) for TLCBD and can be denoted as:

$$[A] = \begin{bmatrix} [0]_{(3,3)} & \{0\}_{(3,1)} & [I]_{(3,3)} & \{0\}_{(3,1)} \\ \{0\}_{(1,3)} & 0 & \{0\}_{(1,3)} & 1 \\ [M]^{-1}_{(3,3)}[K]_{(3,3)} & \omega_g^2\{r\}_{(3,1)} & [M]^{-1}_{(3,3)}[C]_{(3,3)} & 2\omega_g\xi_g\{r\}_{(3,1)} \\ \{0\}_{(1,3)} & -\omega_g^2 & \{0\}_{(1,3)} & -2\omega_g\xi_g \end{bmatrix}$$

In the stochastic analysis, normally instead of the direct responses, the covariances of the responses are evaluated.

The response covariance matrix can be obtained by assuming the stochastic structural process to be Markovian[20]

$$[A][R]+[R][A]^T+[S_{ww}]=\frac{d}{dt}[R] \qquad (10)$$

The matrix $[S_{ww}]$ are matrices of size (8 x 8) for TLCBD which can be expressed as all the terms zero in the matrix except the last diagonal term $2\pi S_0$. Here $[R]$ is the covariance matrix of size (8 x 8).

The response statistics (velocity, acceleration) of the derivative process can be obtained from the above equation. The RMSD can be obtained by using 4th order Runge-Kutta integration method from the above covariance response matrix for TLCBD-structure system as follows:

$$\sigma_{z_3}=\sqrt{R(3,3)} \qquad (11)$$

$$\sigma_{z_2}=\sqrt{R(2,2)} \qquad (12)$$

where σ_{z_3} is representing the RMSD of the structure and σ_{z_2} is representing the RMSD of damper correspondingly. Velocity and acceleration of the system can be evaluated by using the derivative process. In the same manner, the stochastic response of the TLCD-structure system shown in figure 1b under stochastic earthquake can be evaluated but not shown here. More detailed derivation can be found in other literatures[26,29].

Reliability-based optimisation of damper parameters: Usually, transforming the conventional SSO of the damping system into standard nonlinear programming problem is by counting the response of the basic structure as the objective function. In this regard, the RMSD does not properly resemble the reliability criteria for design. To accomplish the desired level of reliability design of the liquid dampers, it is profoundly needed to minimise the structural performance in terms of strength of serviceability criteria in the limit state. In this problem, the objective function is the failure probability of the primary structure. Optimization of TLCD system requires determining the optimum γ_l and head loss coefficient, ϕ. For TLCBD system the optimised parameters are γ_b and R_{12}.

Usually, it has been observed that for the optimal value of γ_l and ϕ of the TLCD or γ_b and R_{12} of TLCBD, the structure attains the minimum displacement. Thus, the design vectors can be defined as $d_{(TLCD)}=(\gamma_l,\phi)$ and $d_{(TLCBD)}=(\gamma_b,R_{12})$.

The reliability of a structure for a given period $[0,T]$ is determined by its failure probability P_f, which can be assessed by the exceedances of lateral displacement to a given threshold value β for the primary structure considered[5].

$$P_f=1-\exp\left[-\eta_\beta T\right] \qquad (13)$$

$$\eta_\beta=\frac{1}{\pi\sigma_{z_3}}\sigma_{\dot{z}_3}\exp\left(-\frac{\beta^2}{2\sigma_{z_3}^2}\right) \qquad (14)$$

By minimizing the failure probability of the seismic excited primary structure, the optimum TLCBD and TLCD parameters can be obtained.

Find $d_{(TLCD)}=(\gamma_l,\phi)$ and $d_{(TLCBD)}=(\gamma_b,R_{12})$ to minimize $F_0=P_f$

$$P_f=1-\exp\left[-\left\{\frac{1}{\pi\sigma_{z_3}}\sigma_{\dot{z}_3}\exp\left(-\frac{\beta^2}{2\sigma_{z_3}^2}\right)\right\}T\right] \qquad (15)$$

The objective function P_f is a function of structure and damper parameters. These parameters are taken as deterministic while solving the stochastic optimization problem and thus optimum TLCD and TLCBD parameters found out are conditional. Duration of the earthquake has been denoted by T. By using the available optimization algorithms present in MATLAB toolbox for gradient-based standard nonlinear optimization, these kinds of nonlinear unconstrained optimisation problems can be solved.

Numerical study: An SDOF system with attached TLCBD on it representing the primary structure has been represented in figure 1a. Both the damper-structure systems are subjected to stochastic earthquake excitation considered to evaluate the effectiveness in terms of failure probability. In this numerical study, the particular focus is to calculate the optimum parameters and its performance for mitigation of the structural response. The following values are assumed unless mentioned otherwise for the present numerical study.

For both the TLCD and TLCBD, various mass ratios of the dampers have been considered and the optimised tuning ratio is plotted against them in figure 2. 1% to 7% mass ratios have been considered here to study the various optimisation parameters. The optimum ϕ for TLCD and the optimum R_{12} for TLCBD have been presented in figure 3 and figure 4 respectively. Optimised value of R_{12} for TLCBD decreases with the increase in the mass ratio of TLCBD and optimum ϕ increases with the same.

The increment of tuning ratio up to a certain limit, which is close enough to match with the frequency of the structure, increases the efficiency of the TLCBD. As the optimum ball tube diameter ratio decreases, it allows more liquid to pass through the tube, which increase the effective mass of the liquid taking part in response mitigation and hence reduces the probability of failure.

Table 1
Properties of damper, structure and earthquake

Properties of dampers	TLCBD	TLCD
μ	3%	3%
p	0.75	0.75
ρ_l (Kg/m^3)	1000	1000
ρ_b (Kg/m^3)	7500	-
ν (Nm/sec)	0.001	0.001
Time period of the structure = 1.3 sec, ξ_s = 2%, $\beta = 2\sigma_{z_3}$, ω_g = 9π rad/sec, ξ_g = 0.6, T = 20 sec, S_0 = 0.03 m^2/sec^3.		

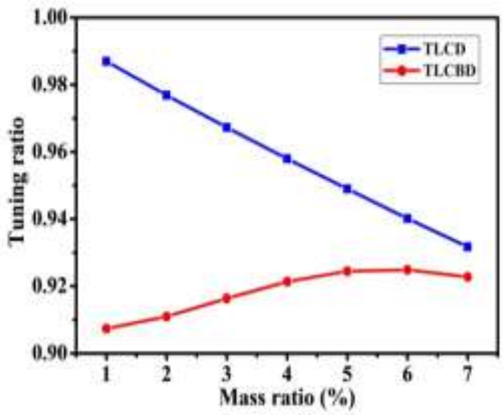

Figure 2: The optimised tuning ratios with different values of mass ratios

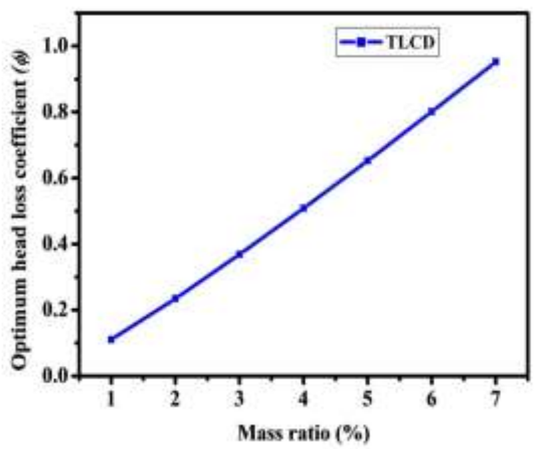

Figure 3: The optimised head loss coefficient for TLCD with different values of mass ratios

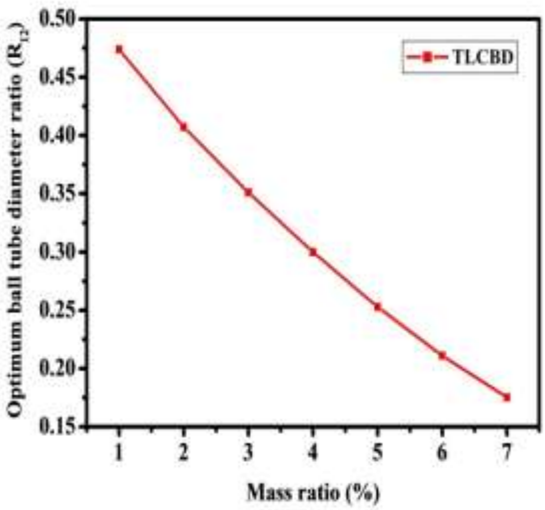

Figure 4: The optimised ball tube diameter ratio for TLCBD with different values of mass ratios

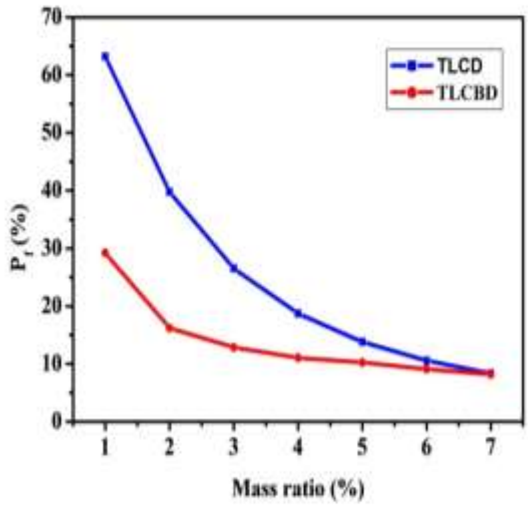

Figure 5: Failure probability of structure with different values of mass ratios

The comparison of failure probability for both the cases is plotted in figure 5 which depicts the better performance of TLCBD over TLCD with the uprising mass ratio.

Due to the combination of both the improved γ_b and R_{12} with increasing mass ratio, TLCBD performs in more effective way than the TLCDs. With the increasing mass ratio, the R_{12} decreases and the tuning ratio for both the dampers tend to merge with each other. As a result, for the higher mass ratio of the TLCBD, it starts acting like a normal TLCD. Generally higher mass ratios were not considered to control the probability failure of structures due to the extended load-carrying capacity and decreased serviceability of the structure which result in increased cost of it. Usually 2% to 5% mass ratios are being adopted in case of designing any passive damper system and in this context TLCBD is found better to provide greater safety and reliability of the structure.

The failure probability of structure decreases for both types of dampers with increasing length ratio. To mitigate the response property of a structure, the TLCBD is found much effective than TLCD for the same characteristic parameters that provide better reliability. It can be verified from the results plotted in figure 9. The corresponding tuning ratio has been shown in figure 6.

The other optimal parameters, for TLCD the ϕ and for TLCBD R_{12} have been plotted in figure 7 and figure 8 respectively. The same amount of efficiency can be achieved with lower length ratio of TLCBD compared to the TLCD. As a result of this phenomenon, the material used to construct the damper becomes less and turns out to be a more cost-effective system.

In figure 10, the trend of optimum tuning ratio for both the dampers has been plotted and compared. For TLCD, the determining optimal parameter is head loss coefficient which is shown in figure 11 for various structural damping ratios. With the increase in structural damping ratios, the head loss coefficient also gets improved to maintain efficiency.

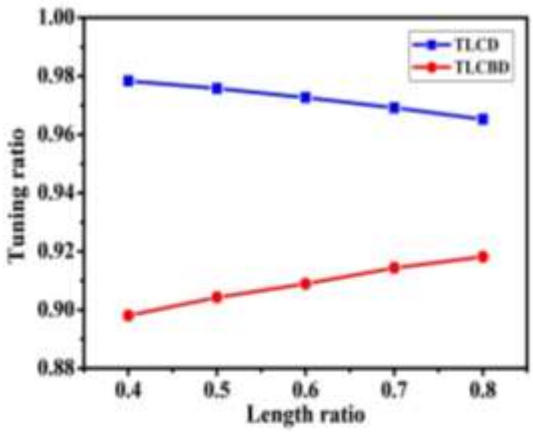

Figure 6: The optimised tuning ratios with different values of length ratios

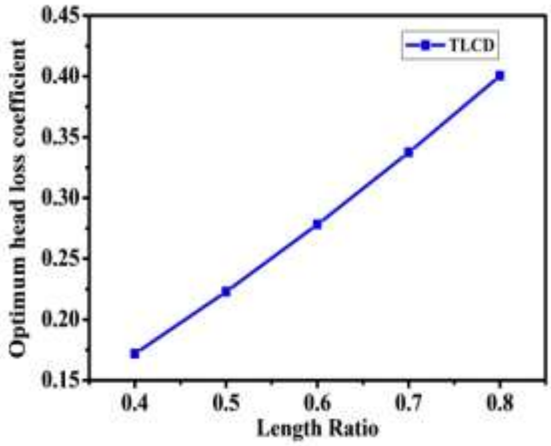

Figure 7: The optimised head loss coefficient for TLCD with different values of length ratios

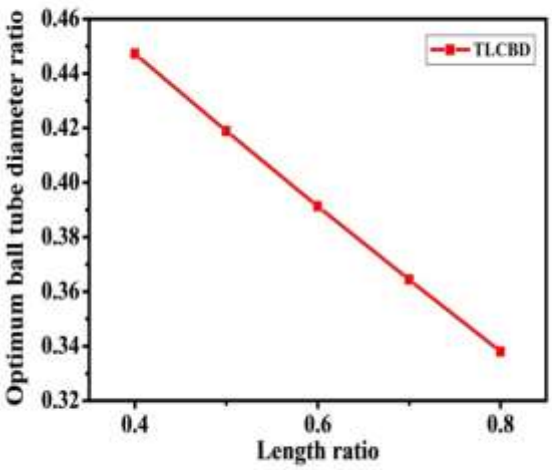

Figure 8: The optimum ball tube diameter ratio for TLCBD with different values of length ratios

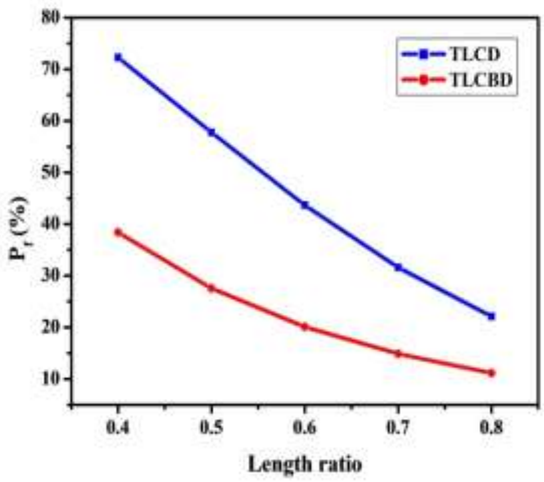

Figure 9: The failure probability of structure with different values of length ratios

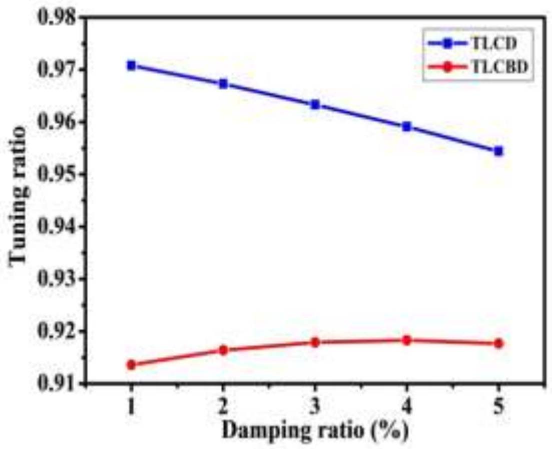

Figure 10: The optimised tuning ratios with different values of damping ratios of structure

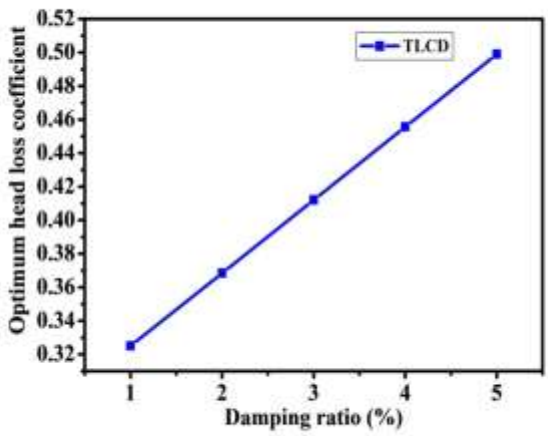

Figure 11: The optimised head loss coefficient for TLCD with different values of damping ratios of structure

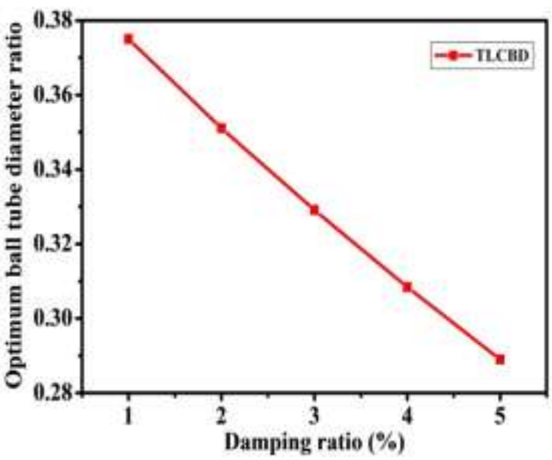

Figure 12: The optimum ball tube diameter ratio for TLCBD with different values of damping ratios of structure

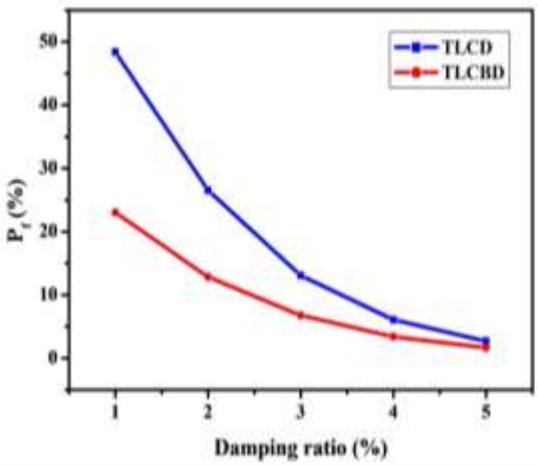

Figure 13: The failure probability of structure with different values of damping ratios of structure

The optimum R_{12} of TLCBD is plotted against different ξ_s and shown in figure 12. It can be noted that the optimised value of R_{12} has a tendency of getting decremented with the increasing value of ξ_s, which allows the higher mass of liquid to contribute in response mitigation and hence, reduces the corresponding probability of failure. Comparison of P_f for both the cases is shown in figure 13 and the results verified the better performance of ball damper over the conventional liquid column dampers. With the increase in ξ_s, the maximum response mitigation is done by the damping property of structure itself. Therefore, the efficiency of damper decreases with the increasing value of ξ_s.

The TLCBD is more effective than TLCD in case of increase in seismic vibration also. Figure 17 shows a comparative analysis for both the dampers. Comparison of change in tuning ratio with varying power spectral density is shown in figure 14. The corresponding other parameters for both the dampers are plotted on figure 15 and 16 respectively. For lesser values of seismic excitation, the vibration mitigation is taken care of by structure itself so the failure probability is approximately the same using both the dampers. But as the intensity increases, it has been noticed that the TLCBD performs better than the TLCD and the effect of structural damping becomes insignificant.

Conclusion

A reliability-based optimization study of a structure attached with TLCBD and TLCD subjected to a stochastic earthquake has been performed considering deterministic system parameters. In this regard, the probability of failure has evaluated considering first passage failure theory.

The probability failure of structure is considered as an objective function here for two different systems (TLCBD and TLCD). For different structure and damper parameters like mass ratio, length ratio, damping ratio of structures and earthquake intensity, failure values are compared for both the systems.

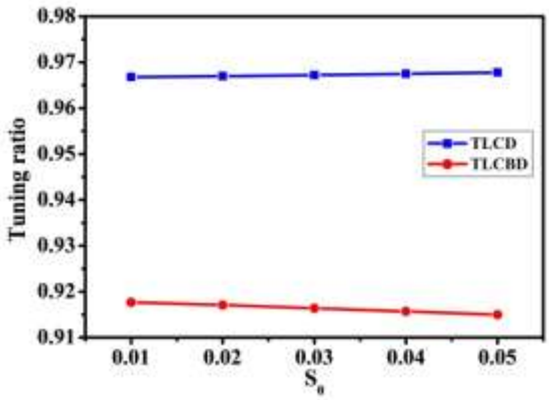

Figure 14: The optimised tuning ratios with different values of S_0

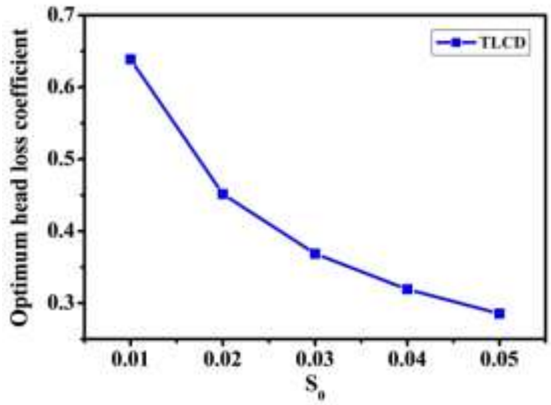

Figure 15: The optimised head loss coefficient of TLCD with different values of S_0

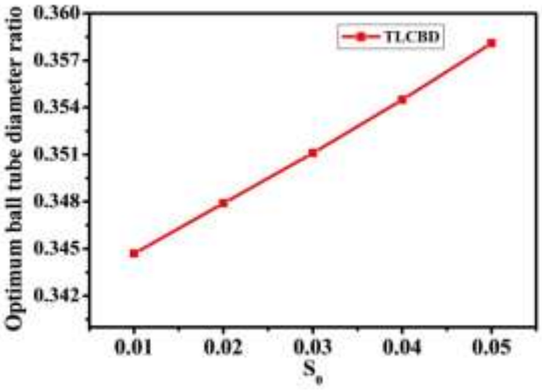

Figure 16: The optimum ball tube diameter ratio for TLCBD with different values of S_0

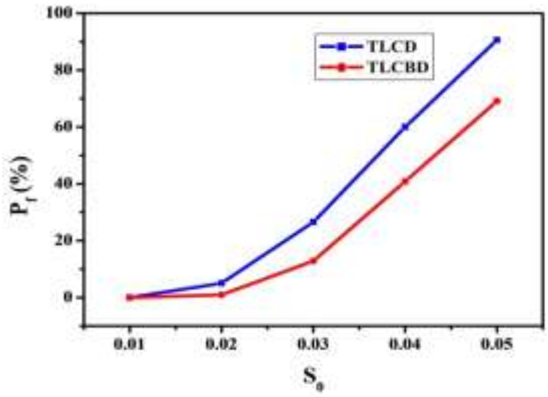

Figure 17: The failure probability of structure with different values of S_0

It is also prominent from the study that the optimum tuning ratio is less in the case of TLCBD system compared to the TLCD system. Even if, the pattern of the optimization results of probability failure obtained from both the systems is alike but in every case, it can be shown that structure with TLCBD system has provided better safety (less probability of failure) than the structure with TLCD system. So from the risk point of view, it can be concluded that structure with TLCBD system provides better efficiency than the other case. This justifies the necessities of RBDO of these two damper systems.

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