Mathematical Modelling to study locally the Seasonal Variability of Plankton Dynamics and Forage Fish in shallow Lagoon using Nutrient Phytoplankton Zooplankton and Forage Fish Model: Chilika - a case study

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Abstract

This study deals with the numerical simulation of the variations of plankton and forage fish in Chilika lagoon (19°28′N-19°54′N and 85°06′E-85°36′E), the largest shallow water lagoon in India. Seasonal variability of plankton and forage fish in the Chilika lagoon is studied with a four-compartment ecological model i.e. Nutrient, Phytoplankton, Zooplankton and Forage Fish. Numerical simulations for each sector of the lagoon are carried out with differential equations for each of the variables. Parameterized values for twenty-two unknown variables are used in the formulation of the differential equations. The values of these variables are found by using sensitivity analysis.

Model results are validated qualitatively with the available data for each sector of the lagoon and this verification with the observations shows a reasonably good match. Most of the significant characteristics of the plankton dynamics of the lagoon like bimodal oscillations and major plankton peaks are well captured by the model.

Keywords: Chilika lagoon, Nutrient Phytoplankton Zooplankton and Forage Fish model, Sensitivity analysis.

Introduction

Shallow coastal lagoons are home to a rich variety of species such as algae, plankton, benthos and fish. These nutrient rich species accumulate a lot of anthropogenic pollutants which make these lagoons highly productive for aquaculture projects and fisheries. Chilika lagoon (19°28′-19°54′N; 85°06′-85°35′E), in the State of Odisha in India, is the largest brackish water lagoon in India and the second largest in the world. This site is listed as a tentative United Nations Educational, Scientific and Cultural Organisation (UNESCO) World Heritage location. Chilika is a shallow water lagoon which has a congregation of fresh water, brackish water and marine components. This lagoon is an ecosystem with large fishery resources that provide livelihood to the local population. The lagoon faces several threats like shrinkage of water surface area due to excessive siltation and choking of the inlet connecting to the Bay of Bengal which has led to an overall decrease in the salinity and ultimately the fishery resources.

There has been an overall loss of biodiversity with decline in productivity which has negatively affected the livelihood of the fishermen dependent on this lagoon. It is necessary to study the effects of environmental conditions on the ecosystem of this lagoon and for that purpose we have developed a basic numerical model containing equations for nutrients, phytoplankton (microscopic plants found on surface of aquatic body), zooplankton (microscopic animals found in aquatic systems) and forage fish.

In the past few decades, scientists and researchers from different disciplines have carried out research in the dynamics of the marine ecosystems. A number of different models are available in literature to study plankton dynamics and they have been successfully used as predictors of marine ecosystem[6-10,13,14,22,32]. One of the simplest and conventional ecological models based on three compartments (*NPZ*) has proved to be a successful model to study the dynamics of a marine ecosystems[4-10,14,15,23,29,32].

Travers et al[33] observed that biogeochemical models cannot be used to assess the effects of fishing on the marine environment. Forage fishes are small, short lived, pelagic fish but they play crucial role in a marine food web. End-to-end mathematical models, which represent the effects of interactions between the physical environment and living organisms from the lower to higher tropic levels, are more appropriate for such studies[3,11,12,18,28].

Naithani et al[24] applied the eco-hydrodynamic model in Lake Tanganyika (03°20′S-08°48′S, 29°03′E-31°12′E) to study the productivity of plankton during the transition between two seasons (beginnings of dry and wet seasons). Travers et al[34] developed an end-to-end model by coupling two existing models to describe the temporal variations of phytoplankton and zooplankton belonging to low and high tropic levels. Turners et al[35] observed that *NPZ* models must be extended to include additional interactions, may be with benthos or fish. Ghosh and Kar[16] studied a prey-predator dynamics model by considering alternative food web when the logistic growth rate is low. Kumar and Kumari[19] used *NPZF*, a non-linear mathematical model to study the dynamics of plankton and forage fish in the Gulf of Kachchh (22°20′N-23° 40′N, 68°20′E-70°40′E).

The non-linear mathematical model Nutrient, Phytoplankton, Zooplankton and Forage Fish (*NPZF*: Figure 1) is adopted in the present study to assess the effect of additional dependent variable, forage fish, on plankton dynamics in the lagoon. The model consists of four different equations and each equation represents the evolution of one state variable with time. Since the model is highly non-linear, very small change in the parameter can lead to the system unstable. The range of suitable parameters in this study is estimated by performing a sensitivity analysis on the model. During sensitivity analysis it is observed that some of the parameters are very sensitive to the model initial conditions. The sector wise (Study Area) observation of each state variable is obtained over the course of the year of available data.

It is also observed that the model is able to produce peaks of some months qualitatively. For better understanding of the stability of the steady state of a dynamical system, phase space analysis is also performed. Due to the addition of extra component forage fish, the unknown parameters involved in the equation play a key role for the stability of the model. The model is successfully applied across sectors of the lagoon and is able to get the desired results which are also compared with the data qualitatively available[1,31].

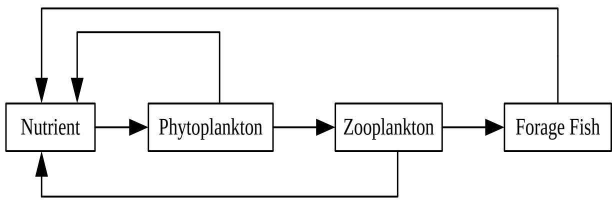

Figure 1: Schematic diagram of the model NPZF in ecosystem

In this study, mathematical formulation of the model which elaborates the governing equations of the model and the parameters associated with it. The area and key characteristics of the case study are described in study area. Sensitivity analysis and phase space analysis section of the model describe the general behavior of the model and to give support to the results obtained during sensitivity analysis.

Mathematical formulation of the model: The general formulation of the governing equations is the same as given by Kumar et al[19] and Panda et al[25]. The equations that give the time evolution of a chemical or biological quantity are given by:

$$\frac{dC_i}{dt} = S_i + D_i \qquad (1)$$

where S_i and D_i represent the source and decay terms of the i^{th} biological or chemical tracer of the concentration C_i where $i = 1,2,3.........$

Based on this, the governing equations representing the four compartments *NPZF* model are as follows:

$$\frac{dN}{dt} = -\left\{\frac{\alpha(\varphi,H,t)N}{K_N+N} - r\right\}P + \frac{m_0}{H}(N_0 - N) \qquad (2)$$

$$\frac{dP}{dt} = \left\{\frac{\alpha(\varphi,H,t)N}{K_N+N} - r\right\}P - \frac{c_0(P-P_0)}{K_Z+(P-P_0)}Z - \frac{q_1 P}{B}\frac{c_1(B-B_0)}{K_F+(B-B_0)}F - \frac{m_1 P}{H} \qquad (3)$$

$$\frac{dZ}{dt} = \frac{e_0 c_0(P-P_0)}{K_Z+(P-P_0)}Z - g_1 Z - \frac{q_2 Z}{B}\frac{c_1(B-B_0)}{K_F+(B-B_0)}F \qquad (4)$$

$$\frac{dF}{dt} = \frac{e_1 c_1(B-B_0)}{K_F+(B-B_0)}F - g_2 F \qquad (5)$$

where $B = q_1 P + q_2 Z$, denotes the total amount of food consumed by the forage fish.

The units and ranges of the parameters used in this study are given in table 1. Units of *N*, *P*, *Z* and *F* for simulation are taken in *mg/l* and time unit is in days. The parameters used in governing equations 1 - 5 are explained below:

Nutrient: Lagoons are highly productive ecosystems due to the input of a large amount of nutrients through sedimentation and anthropogenic sources. The nutrients are depleted due to the uptake by phytoplankton and settling down.

Equation (2) represents the growth rate of nutrients over time *t*. The first term $-\left\{\frac{\alpha(\varphi,H,t)N}{K_N+N}\right\}$ represents the reduction in the concentration of nutrients due to daily specific growth rate of phytoplankton which is represented by a Holling type II[17] functional response.

The term $\alpha(\varphi,H,t)$ represents the light limited growth rate of phytoplankton (photosynthetic rate of phytoplankton[9]) and $\frac{N}{K_N+N}$ represents the nutrient limiting factor with half saturation constant K_N and is governed by Michalis-Menten (M-M) formula[21]. This is a parameter which needs to be estimated using the sensitivity analysis. The term *rP* denotes the addition to the nutrient concentration due to phytoplankton mortality.

The term $\frac{m_0}{H}N_0$ denotes the increase in nutrient concentration through vertical diffusion from the sediments at depth H where N_o is the sediment nutrient concentration. This is due to the shallow nature of the lagoon which makes the mixing from the lagoon floor possible. The loss in nutrients due to the settling of the nutrients to the lagoon floor is represented by $-\frac{m_0}{H}N$; m_0 in this equation is the diffusion coefficient.

Photosynthetic rate (α): The growth rate of phytoplankton is averaged over the course of a day[5,9,19,25] as follows:

$$\propto(\varphi,H,t) = \frac{2Q}{k1H}\int_0^{\tau}\int_{\beta}^{\beta e^{k_1H}}\frac{tdydt}{y(y^2+t^2)^{1/2}}$$

where $\beta = \frac{Q\tau}{k_2J}$, $\tau = \left(\frac{1}{2}\right)\arccos(-\tan\delta\tan\varphi)$, δ is the declination and φ is the latitude

$$J\tau = \left(\frac{R}{\pi}\right)(\tau\sin\delta\sin\varphi + \cos\delta\cos\varphi\sin\tau)$$

$$R = \left(\frac{3}{8}\right)(1-a_0)S_0$$

where a_0 is the average albedo of Earth and R = 1.375kWm^{-2} is the solar constant.

Table 1
Description of parameters along its values and units used in the model

Parameter description	Symbols (Unit)	Assigned values at Northern sector	Assigned values at Central sector	Assigned values at Southern sector
Half saturation coefficient of Phytoplankton	$K_N(mg/l)$	1.5	6.5	2.5
Mortality loss rate of Phytoplankton	$r(d^{-1})$	0.0507	0.04	0.075
Depth of the Chilika lagoon	$H(m)$	2.5	3	2
Vertical mixing rate	$m_0(md^{-1})$	2.0	2.5	1.5
Nutrient source	$N_0(mg/l)$	100.25	100.25	100.25
Threshold value of phytoplankton	P_0	40.0	15	40
Half saturation coefficient of Zooplankton	$K_Z(mg/l)$	150	170	180
Maximum grazing rate of Zooplankton	$c_0(d^{-1})$	0.3	0.3	0.4
Palatability coefficient of Zooplankton	q_1	5	5	0.48
Maximum grazing rate of Forage fish	$c_1(d^{-1})$	0.1	0.2	0.2
Threshold value of B	$B_0(mg/l)$	20	10	10
Half saturation coefficient of Forage fish	$K_F(mg/l)$	70	90	90
Settling rate of Phytoplankton	$m_1\ (md^{-1})$	0.8	0.7	0.35
Grazing efficiency of Zooplankton	e_0	0.44	0.54	0.45
Mortality rate of Zooplankton	$g_1(d^{-1})$	0.01	0.01	0.015
Palatability coefficient of Forage fish	q_2	0.02	0.02	0.02
Predation efficiency of Forage fish	e_1	0.1	0.1	0.2
Mortality rate of Forage fish	$g_2(d^{-1})$	0.0052	0.0052	0.0061

Phytoplankton: Equation (3) describes evolution of phytoplankton with time *t*. The first term $\frac{\alpha(\varphi,H,t)N}{K_N+N}$ represents the increase of phytoplankton density by taking up nutrients (explained in Nutrient) and *-rp* represents the decrease of their concentration due to the natural mortality as well as the losses due to the process of respiration which utilizes the energy obtained during photosynthesis. The term $\frac{c_0(P-P_0)}{K_Z+(P-P_0)}Z$ represents the loss of phytoplankton due to consumption by zooplankton and this equation is obtained again using the M-M equation and K_z as the Michealis constant which needs to be estimated. C_0 denotes the maximum grazing rate of the zooplankton.

A threshold value P_0 is introduced in this term to ensure that the phytoplankton does not become 0. The decrease in phytoplankton due to the feeding of forage fish is represented in the fourth term:

$$\frac{q_1 p}{B}\frac{c_1(B-B_0)F}{K_F+(B-B_0)}$$

where $B=q_1P+q_2Z$, denotes the total amount of food consumed by the forage fish. q_1 and q_2 represent the palatability coefficients for phytoplankton and zooplankton respectively which indicate the preference of food by the forage fish[20,24]. This term is once again formulated using the M-M equations with K_F as the Michealis constant. The last term $\frac{m_1P}{H}$ in equation 3 denotes the loss of phytoplankton due to settling to the lagoon floor with settling coefficient m_1 which needs to be estimated.

Zooplankton: Equation (4) denotes the rate of change of zooplankton density over time *t*. The term $\frac{e_0c_0(P-P_0)}{K_Z+(P-P_0)}Z$ denotes zooplankton growth due to consumption of phytoplankton with half saturation parameter K_z.e_0 is the efficiency coefficient which denotes the amount of food consumed by zooplankton that is actually converted to energy (the rest is excreted as waste). The second term in this equation denotes the decrease in zooplankton by natural mortality with mortality coefficient g_1 which is to be determined.

The term $\frac{q_2Z}{B}\frac{c_1(B-B_0)F}{K_F+(B-B_0)}$ represents the reduction of zooplankton population due to predation by forage fish formed by using the M-M equation as in the previous equations.

Forage Fish: Forage fishes are small, short lived, pelagic fish and are primary food source of many marine mammals (larger predators). They are also important predators in a marine ecosystem and prey upon phytoplankton and zooplankton.

In the governing equation, equation (5) represents the rate equation for changes in fish concentration over time t. The concentration increases because of the predation of fish on zooplankton and grazing on phytoplankton, this is represented by the term $\frac{e_1c_1(B-B_0)}{K_F+(B-B_0)}F$, a type I functional response[24] (with assimilation efficiency e_1 and predation rate C_1).

Parameter e_1 represents the efficiency coefficient of the fish and as in the case of zooplankton, it denotes the amount of food converted to energy. The fish population density decreases due to the mortality loss g_2F where g_2 is the mortality coefficient whose value will be estimated by sensitivity analysis.

Study Area - Chilika Lagoon

Chilika lagoon (www.chilika.com) is located in the eastern part of India in the State of Odisha (Figure 2). The lagoon is known for its mesmerizing beauty with several islands, a bird sanctuary and a unique biodiversity. The rich fishery resources of the lake provide livelihood opportunities to the fishermen. The size of the lagoon sways markedly within the period of a year, ranging from a maximum area of 1,165 km^2 during monsoon to a minimum of 906 km^2 during dry season[2,30].

The combined inflows of fresh, brackish and saline water make the environment of the lagoon exceptionally productive. The important hydrological influences on the lagoon include fresh water and silt influx from river Mahanadi and its tributaries, exchange of marine water with Bay of Bengal and monsoon lead rainfall.

All these contribute to the drainage basin area of the lagoon. The entire lagoon is separated into four zones namely, northern, central, southern sectors and the outer channel. A 32 km long outer channel connects the lagoon with the Bay of Bengal (Figure 2).

In 2000, the Chilika Development Authority (CDA) opened a new “mouth” connecting the lagoon with the Bay of Bengal. Due to siltation, the width of the mouth gets fluctuated periodically and shifted from place to place which affects the fishermen and they had to change their occupation for survival.

Plankton and Nutrient characteristics in Chilika Lagoon: The environment of Chilika lagoon is extremely complex which makes it difficult to model the temporal and seasonal variations of plankton. This is also due to the morphological conditions at different sectors of the lagoon and at different seasons.

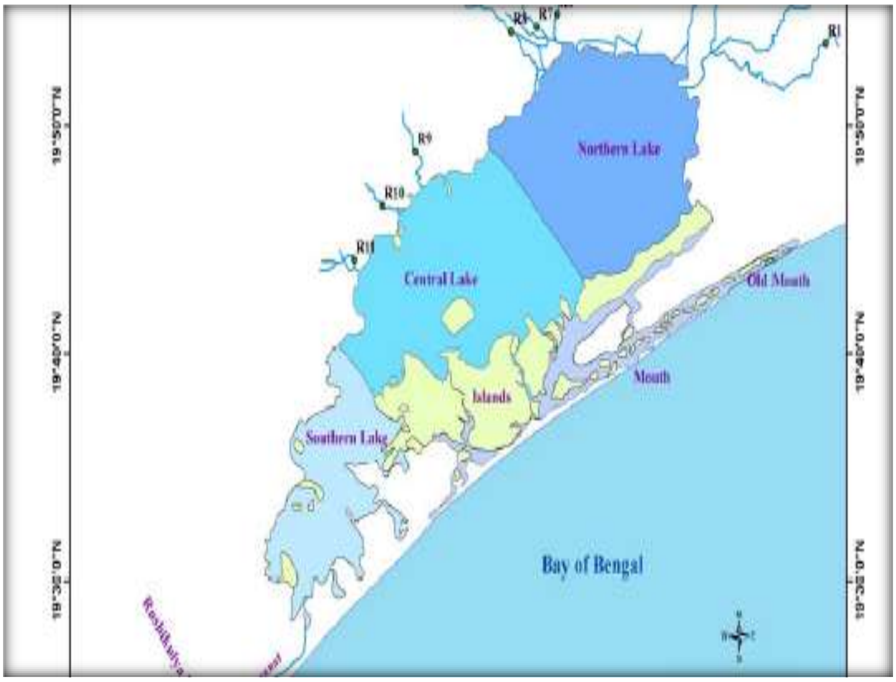

Figure 2: The Chilika Lagoon

Due to a large inflow of fresh water through various tributaries, the northern sector of the lagoon shows high water nutrient content during summer. Panda and Mohanty[26] using remote sensing data observed the various parameters of water quality from 2001-2006 at different stations of Chilika lagoon and observed that the nutrient concentration in the Northern sector is high in summer where as in winter, the central sector dominates in water nutrient content. Similarly, chlorophyll (CHL-A, CHL-B and CHL-C) shows maxima in northern sector during both summer and winter seasons.

Panda and Mohanty[26] also observed that CHL-A contents are comparatively higher in northern sector than the other sectors both in summer and winter. It is also observed that CHL-B and C have a smaller concentration as compared to CHL-A. Panigrahi et al[27] observed the variability of nutrients and phytoplankton biomass in Chilika lagoon during 2001-2003 (from 27 sampling locations). They observed that during monsoon the nutrient concentration is almost double as compared to summer. This maybe attributed due to the heavy rainfall leading to a higher runoff from the surroundings. The CHL-A concentration is higher in summer as compared to post-monsoon and monsoon (JJAS) period.

The observed data of nutrient, phytoplankton and zooplankton at some specific stations for northern, central and southern sectors is given in table 2 and is obtained from Adhikari et al[1] and Srinivasan[31]. Earlier this data was used by Dube et al[5] and Panda et al[29].

The observed data shows that the nutrient concentration is higher in August, September-December due to river runoff and it is lower in April, May and January due to the phytoplankton uptake. The zooplankton distribution is seen to follow the phytoplankton. Since the above data is continuous, it helps in the model comparison. The units of observed and simulated phytoplankton and zooplankton are different, so we can only validate the model qualitatively till better observations for these quantities are available.

Sensitivity Analysis and Phase Space analysis: The values of the parameters play a crucial role in modeling of an ecosystem. Small changes in the values of the parameters result in bigger changes in the dynamics of the system. The small change in value of parameters decides the sensitivity of the model to a particular parameter. For modeling, the sensitivity analysis plays an important role and helps to evaluate the model behavior based on parameter values. Phase space analysis helps to get the better understanding between the result obtained during simulation and the behavior of the model.

Sensitivity Analysis through Numerical Experiments: A sensitivity analysis of parameters involved in the governing equation (2 to 5) is performed to compare the model with the observed data. A good match of the most of the parameters is required for the qualitative comparison of the model results with the observed data given in table 2 and the range of the parameter was determined through sensitivity analysis. During this analysis it was found that the model responses are highly sensitive to some of the parameters where a small change in values can lead to the very different dynamics. Out of twenty two parameters, the five parameters values, maximum growth rate of P (Q), light attenuation parameter (k_1), low light photosynthetic rate (k_2), solar constant (S_0) and average albedo of earth (a_0) are kept constant and taken from the sources[5,19,29].

The system is seen to be sensitive to the rest of the parameters. During simulation, it was observed that the model was very sensitive to the light limited growth rate (α), the half saturation coefficient of $Z(K_z)$, mortality loss of phytoplankton (r), vertical mixing rate (m_0), threshold value of P (P_0), threshold value of B (B_0), palatability coefficient of Z and F (q_1, q_2) and mortality rate of Z and F (g_1, g_2). These parameters also play a very crucial role in model simulation. Phytoplankton population decreases by increasing the half saturation coefficient (K_N), maximum grazing rate of zooplankton (C_0) and also due to the settling rate of phytoplankton (m_1).

It is also observed that by increasing the vertical mixing rate (m_0), the phytoplankton growth rate increases and this is because of the addition of nutrients to the system. The grazing effect of the zooplankton on the system depends on the parameters c_0, e_0, g_1, q_2, c_1, K_Z and K_F. The increase in the value of c_0 leads to the rise of the zooplankton population but it decreases after a particular range, $C_0 > 0.35$, d^{-1} (for Northern sector) due to excess grazing pressure of phytoplankton which leads to the elimination of phytoplankton and causes the decrease of zooplankton population.

The zooplankton will decrease due to the natural mortality, so the population growth decreases as g_1 increases. Similarly, the increase in value of K_Z leads to the decrease in the zooplankton concentration.

The primary focus of this study is to study the impact of forage fish in the model as an improvement of *NPZ* model. The growth of forage fish depends on e_1, c_1, K_F, B_0 and g_2. The fish population will increase by increasing the parameter e_1 and c_1 which lead to the decrease of zooplankton. The model is very sensitive to the parameter g_2 and after a specified range, $g_2 > 0.01$, d^{-1} for (Northern sector) the dynamics of the system changes abruptly. Similarly, increase in the value of K_F leads to decrease in the fish population. So, performing numerical experiments, best values of the parameters are estimated and listed in table 1.

Phase Space Analysis: Phase space diagrams represent the dynamical behaviour of a non-linear system. These diagrams are used to assess the stability of the steady state of a dynamical system. Figures 3 and 4 show the phase space diagrams for *N* Vs *P* and *Z* Vs *P*. These figures show that the diagrams do not converge to a stable steady point.

Instead, they form a closed curve which is known as a limit cycle. A limit cycle is a closed trajectory in space which has a property that at least one other trajectory is seen to spiral into it with increase in integration time. If all the neighbouring trajectories approach the limit cycle (with increase in time) as seen in the figure 3 and 4, the limit cycle is stable or attracting.

On the other hand, if the neighbouring trajectories approach the limit cycle as time reaches negative infinity, then the limit cycle is unstable. This limit cycle corresponds to temporal oscillations which do not dampen with increase in time. This is a typical behaviour for equations representing a biological system.

Table 2
Observed data of Nutrient, Phytoplankton and Zooplankton

Month	Northern Sector			Central Sector			Southern Sector		
	N(µg/l)	P(Nos/l)	Z(ml/l)	N(µg/l)	P(Nos/l)	Z(ml/l)	N(µg/l)	P(Nos/l)	Z(ml/l)
Jan	101.1	400			200			400	
Feb	141.3		2.9	90.4	250		100.31	350	
Mar	111.3	550			200	2	90.6	500	5.8
April	100.8	530	1.5	76.1	300	1.25	86.05		4.7
May	101	300		81.3			85.6	700	
June	150.8	530		86.3	850	1.55	120.65	1300	7.8
July	176.3		1.65	101.1	900		120.95	1600	6.1
Aug	202.1	180	1.3	145.35	600	1.15	121.2	1100	
Sept	196.6	370		145.75			100.7	600	
Oct	196.7	620	1.4	130.9	340		85.55	800	7.7
Nov	180.8	590	1.7	120.7	300	2	80.6	790	6.3
Dec	170.6	300		105.6	300	1.75	90.45	500	

Source: Adhikari et al[1] and Srinivasan[31]

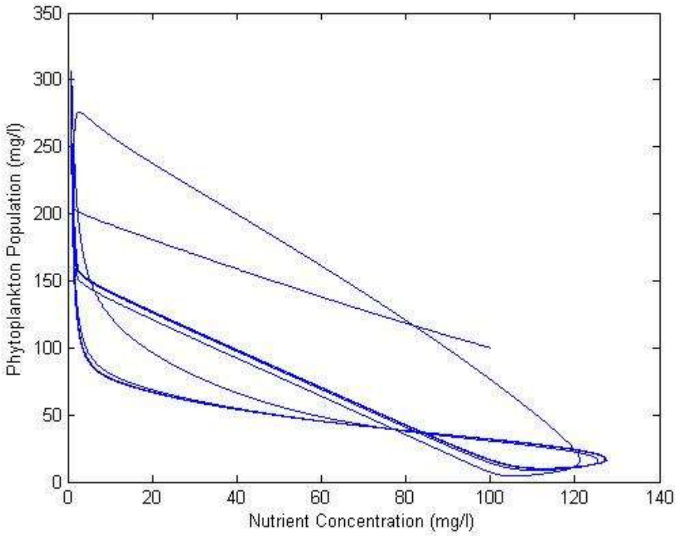

Figure 3: Phase space analysis between Nutrient concentration and Phytoplankton population distribution

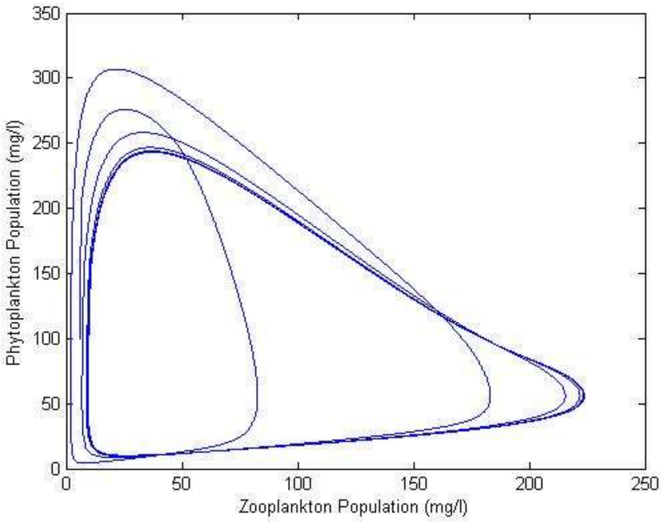

Figure 4: Phase space analysis between Zooplankton and Phytoplankton Population distribution

Results and Discussion

The model is applied in the northern, central and southern sector of the Chilika lagoon. The model simulated results are verified for phytoplankton[1] and zooplankton[31]. Although the units for the phytoplankton and zooplankton are different in the observation and the model forecasts, we can still make a qualitative comparison between the two. This is useful because the model results show that a simple *NPZF* model is capable of predicting the increase and the subsequent decrease in the population of the phytoplankton and the zooplankton reasonably well. This kind of study can help in determining the effect of various factors on the ecosystem of an important region like Chilika lagoon. The data for forage fish is not available, so only the model simulated figure is depicted for different sectors of the lagoon.

Northern Sector: Model results for *N*, *P*, *Z* at northern sector are depicted in figures (5-7) along with the observed data whereas for the forage fish (*F*) (Figure 8), only the model simulated values are shown as the observed values are not available. The observed nutrient concentration[1] shows higher values during August-November and July. Figure 5 shows the model simulated results for nutrients. This figure shows that the model is able to capture some peaks though for the month of October-December, the model is not able to capture the desired peaks. The model simulated results for phytoplankton show that the model is able to capture the peaks in the observations except for the month of March (Figure 6).

The zooplankton distribution is seen to follow the phytoplankton distribution (Figure 7). Since the growth of forage fish depends on the predation of zooplankton and phytoplankton population, the model simulated values (Figure 8) follow the zooplankton. However, due to the non-availability of observed data for forage fish, in the current study, we are presenting only the time evolution of forage fish and its effect on other dependent variables. During sensitivity analysis it was observed that the nutrients play a crucial role in controlling the growth rate of phytoplankton.

The half saturation constant K_N determines the parameters involved in nutrient growth. The range of K_N found suitable for the northern sector is 1-5 and a value less than 0.5 leads to numerical instability in the solution. Another parameter which controls the phytoplankton growth is the mortality loss rate *r*. The range of *r* for the northern sector is 0.05-0.07. Increase in this value above 0.5 leads to an unstable solution. The value of vertical diffusion rate coefficient m_0 is specified as 2.2. Decrease in the value of m_0 results to a fall in phytoplankton production as this determines the amount of nutrients in the system. As expected, increase in the grazing rate of zooplankton leads to the decay of phytoplankton and an increase in the zooplankton population.

The range for C_0 in the northern sector is 0.25-0.35 and a value greater than 0.35 results in decrease of phytoplankton and a rise in zooplankton population. Also, for this value a steady state solution was not obtained. The effect of assimilation efficiency is similar to the grazing rate of zooplankton C_0. The admissible range of e_0 in Northern sector is 0.4-0.9. Decrease in the grazing threshold P_0 results in increase of zooplankton population but after $P_0 > 75$ leads to extinction of zooplankton.

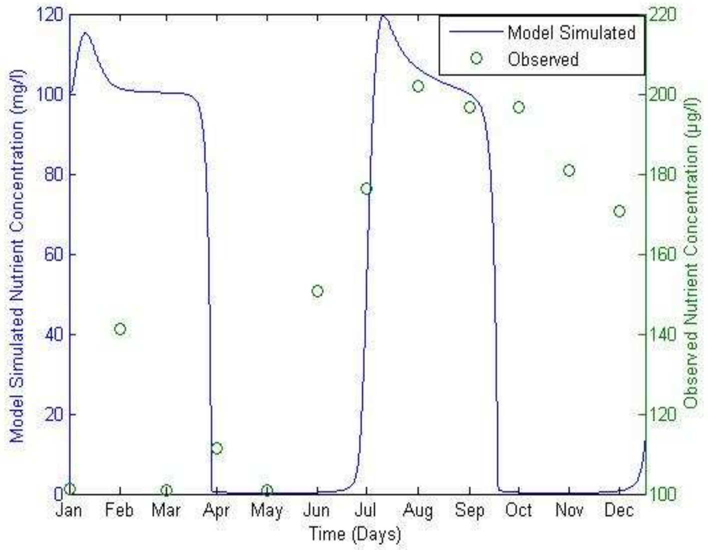

Figure 5: Nutrient concentration distribution at Northern Sector

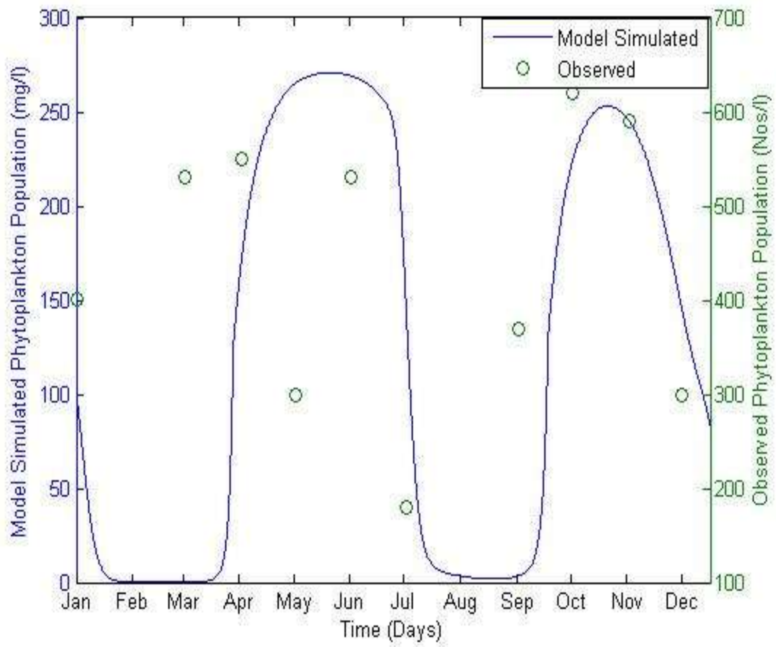

Figure 6: Phytoplankton Population distribution at Northern Sector

The range for P_0 is found in between 30-50 in the northern sector. The system is observed as very sensitive to the parameters involved in the growth of forage fish. The range of mortality rate g_1 of zooplankton is obtained as 0.01-0.03. By increasing the value of g_1, the zooplankton population decreases, also the population of forage fish decreases. A steady state solution cannot be obtained if a value above the specified range is given. The most sensitive parameter for forage fish is its mortality rate.

For northern sector its range is 0.005-0.0055. As this value increases, the forage fish population decreases and for a value greater than 0.01, it leads to the system unstable. The threshold value of B is found to be in the range of 20-30 and for values of B_0 exceeding 39, the concentration of forage fish becomes negative. These values of the parameters are listed in table 2.

Central Sector: The model simulated results along with the observed data for nutrients, phytoplankton and zooplankton are shown in figures 9-11 whereas for forage fish only, model simulated results are shown in figure 12. It is observed that the nutrient concentration is higher in monsoon and post monsoon period (August-October) and the simulated results capture the peaks for the above time period but fail to capture the second peak seen during November (Figure 9). It is seen from observed data that the phytoplankton biomass increases in June and July and the model results reproduce this peak well.

However, it slightly underestimates the peak for the month of August (Figure 10). The observations for zooplankton are also well captured by the model (Figure 11). Regarding the sensitivity, similar behavior of the parameters is observed in both central and northern sector. Figure 12 depicts the forage fish distribution in the central sector and the model is able to obtain the desired peak.

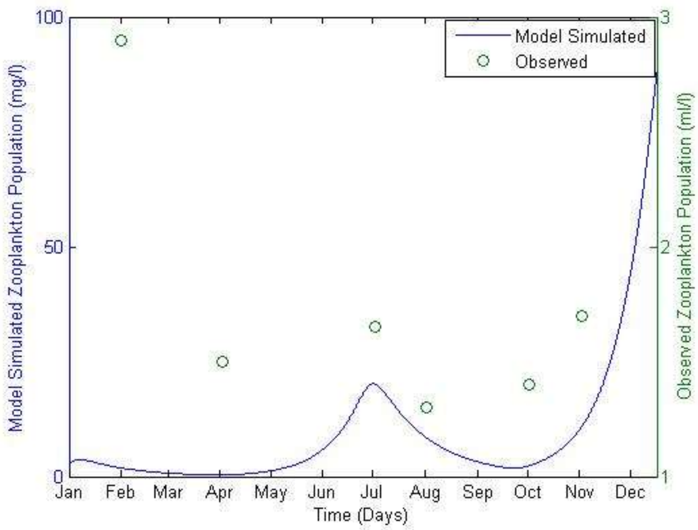

Figure 7: Zooplankton Population distribution at Northern Sector

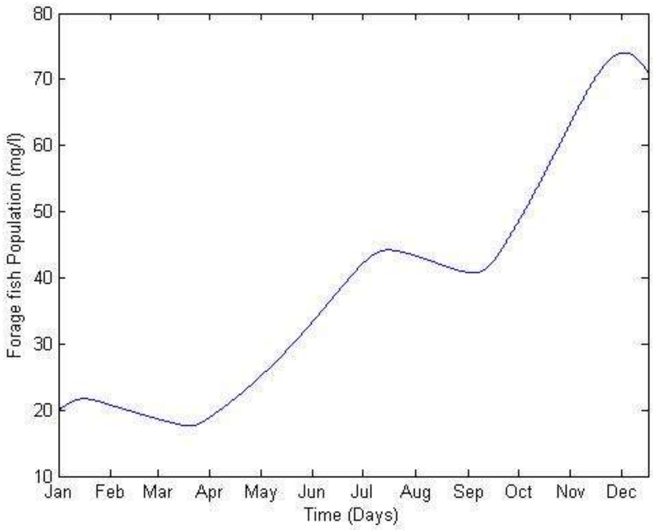

Figure 8: Model simulated Forage fish Population distribution at Northern Sector

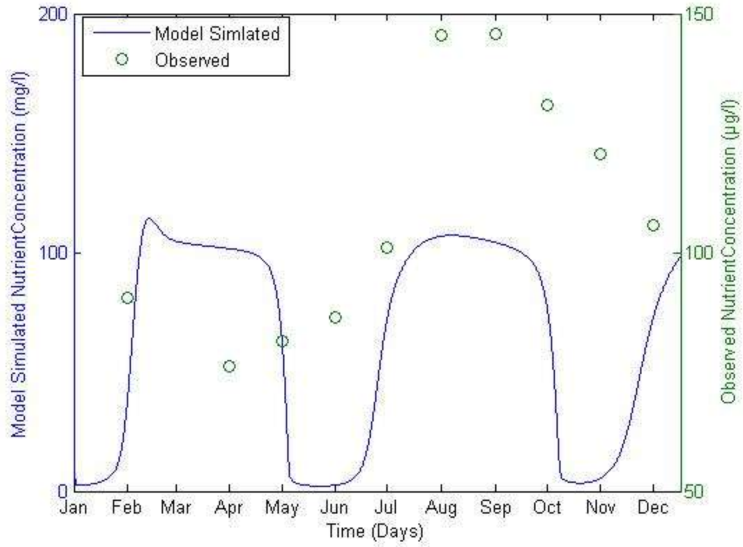

Figure 9: Nutrient Concentration distribution at Central Sector

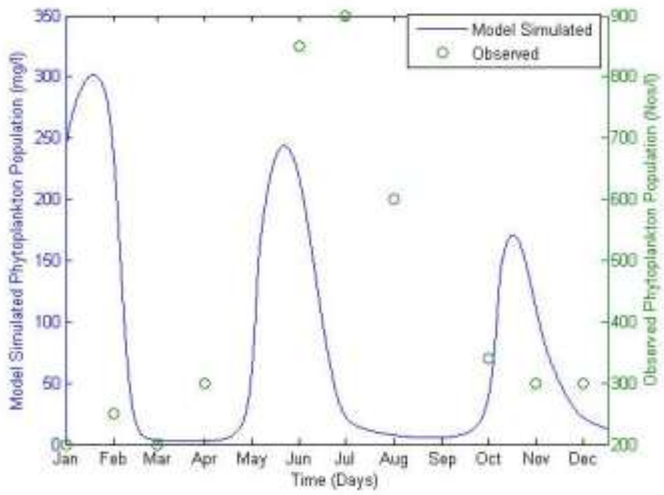

Figure 10: Phytoplankton Population distribution at Central Sector

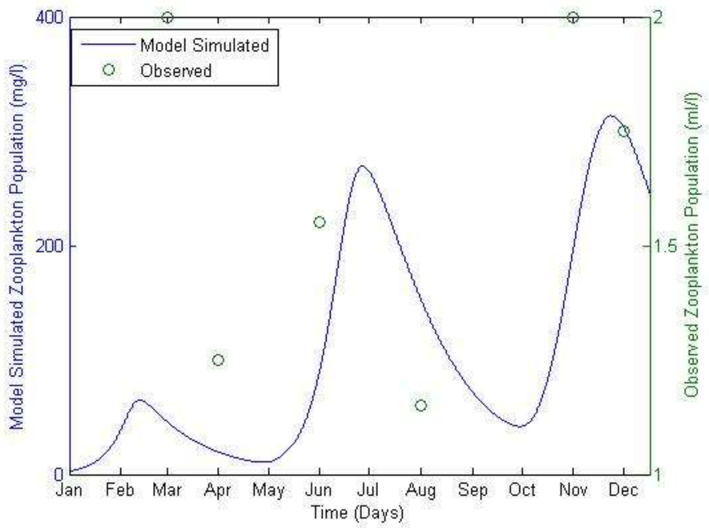

Figure 11: Zooplankton Population distribution at Central Sector

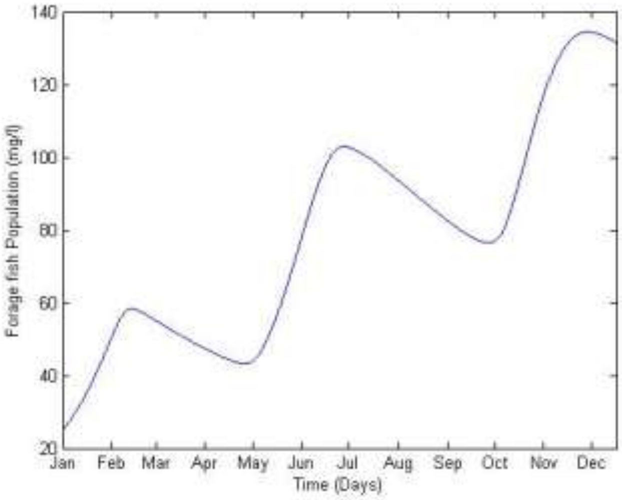

Figure 12: Model Simulated Forage Fish Population distribution at Central Sector

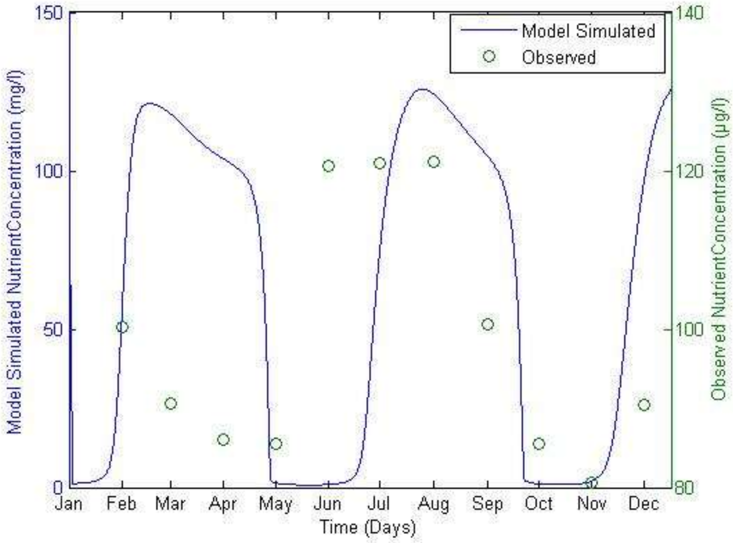

Figure 13: Nutrient Concentration distribution at Southern Sector

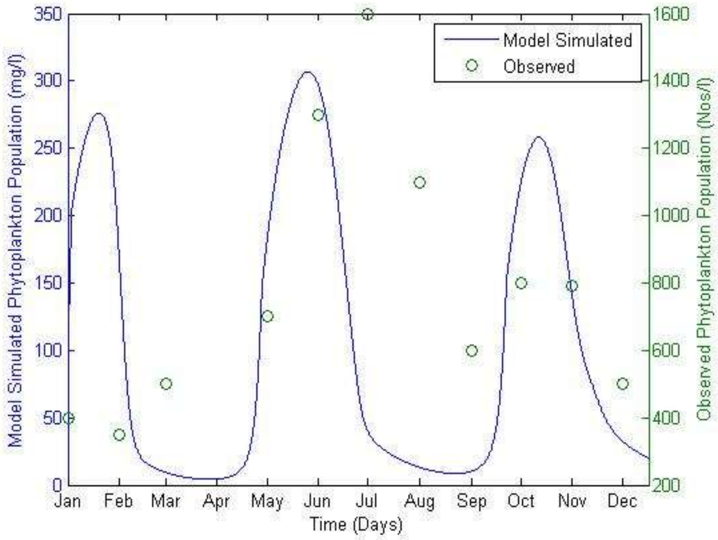

Figure 14: Phytoplankton Population distribution at Southern Sector

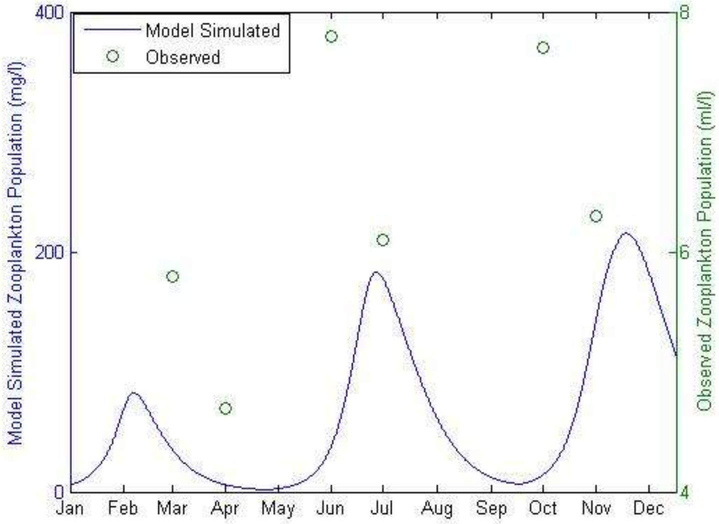

Figure 15: Zooplankton Population distribution at Southern Sector

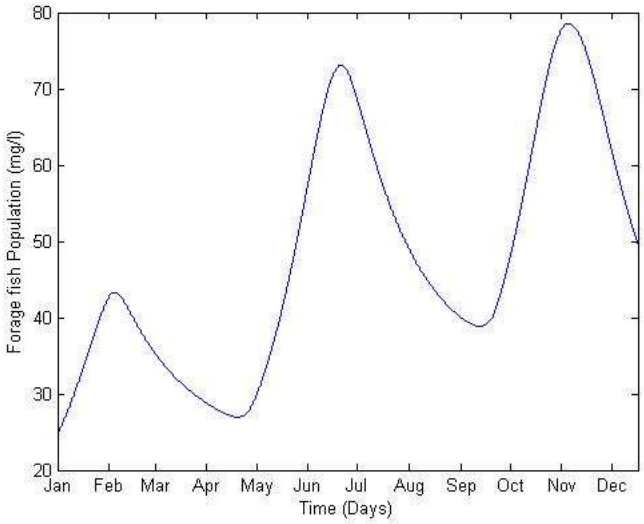

Figure 16: Model Simulated Forage fish Population distribution at Southern Sector

Southern Sector: The southern sector findings are shown in figures 13-16. Figure 13 presents the annual nutrient concentration. It is seen from the figure that the model is once again able to capture the peaks in the observed data. The phytoplankton population (Figure 14) shows that the three peaks in the observations are captured by the model. Figure 15 show that the zooplankton population follows the phytoplankton. Addition of forage fish (Figure 16) shows the changes in the dynamics of the system. The discussion is similar as in the case of northern and central sector.

Conclusion

In the current study, an ecological model is adapted and used to reproduce the variations in concentrations of different species in the Chilika lagoon. A sensitivity analysis is also performed to study the effect of various parameters on the model results. This study shows that the model is very sensitive to some parameters like half saturation coefficients, mortality rates, vertical mixing rate, threshold values and palatability coefficient as compared to other parameters. As seen from the observations, some of the main characteristics like periodicity in phytoplankton, zooplankton and forage fish profiles correlated with the changes in environmental parameters are well reproduced in the model results. The sector wise peaks of nutrient, phytoplankton and zooplankton population are better represented by the model.

However, the model is not able to reproduce the smaller peaks in Central and Southern sectors during the months of March-May and October-November. Model results are compared by using the discrete data available from various sources.

However, a continuous time series data (at least three to four years) will be needed for better comparison and validation. Further analysis including the linear stability of the above defined system which can provide further insight into the behavior of the mathematical model is currently undertaken by the authors.

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